SECTION 5B: EFFECTS OF THE GROUND (INCLUDING GROUND-WAVE PROPAGATION)

REPORT 714-2

GROUND-WAVE PROPAGATION IN AN EXPONENTIAL ATMOSPHERE

(1978-1982-1990)

1. **Introduction**

In the older ground-wave propagation curves which were replaced by Recommendation 368, no account was taken of tropospheric refraction. The troposphere was taken as a vacuum so that its refractive index n and modified refractive index m are:

$$n = 1, \qquad m = n + \frac{h}{a} = 1 + \frac{h}{a} \tag{1}$$

where h is height and a is the Earth's radius.

In the CCIR atlases of ground-wave propagation curves for frequencies above 30 MHz [CCIR, 1955 and 1959], the tropospheric refractive index is taken to decrease linearly with height, as described by:

$$n = n_s - \frac{k-1}{k} \cdot \frac{h}{a}, \qquad m = n_s + \frac{h}{ka} \tag{2}$$

where $n_s \simeq 1.0003$ is the surface value of n, and k is the effective earth radius factor, taken as 4/3 in the curves. Expression (2) encompasses expression (1) with $n_s = k = 1$.

In fact, the troposphere has a mean refractive index which varies exponentially with height as:

$$n = 1 + (n_s - 1) \exp(-h/h_s) \tag{3}$$

where h_s ($\approx$ 7 km) is the scale height of the troposphere. This model is fully described by Bean and Dutton [1966] and accepted as the Reference Atmosphere of Recommendation 369. For $h \gg h_s$ it tends to expression (1). For $h \ll h_s$ it tends to expression (2) with $h_s = ka(n_s - 1)/(k - 1)$.

At frequencies below 10 MHz, the height-gain effects become small at moderate heights and it was partly for this reason that the ground-wave propagation curves have been made to refer only to the case in which both terminals are on the ground. On the other hand, below about 3 MHz, the range of height entering into the determination of the rate of attenuation of field strength with distance around the Earth has extended to the region where the refractive index of the troposphere begins to depart seriously from the value corresponding to a linear decrease with height appropriate to the use of a 4/3 Earth's radius. Thus the rate of attenuation of field strength with distance around the Earth no longer corresponds to the use of an atmosphere in which the refractive index decreases linearly to indefinitely great heights.

While at the upper limit of 10 MHz for the ground-wave propagation curves in Recommendation 368 it is still nearly correct to use an equivalent radius of 4/3 times the real radius of the Earth for both terminals on the ground, the troposphere can have very little effect at the lower limit of 10 kHz, where the range of height entering into the determination of the rate of attenuation of field strength with distance around the Earth extends to many kilometres above the Earth. There is thus a transition that becomes marked at about 3 MHz and almost complete at 10 kHz, from the use of a 4/3 Earth's radius at 10 MHz to the use of the real radius of the Earth at 10 kHz.

2. Theoretical analysis

In recent years a method of analysis has been developed in the United Kingdom [Rotheram, 1970] which enables the effects of a troposphere with an exponential height variation of refractive index to be taken into account. The following sections describe the methods used to calculate the field in various domains.

2.1 *The field near the surface at short ranges*

Close to the transmitter the field near the surface due to a source on the surface can be described by the Sommerfeld flat earth theory. As the range increases corrections due to the effective Earth's curvature can be applied as described by Bremmer [1949]. Further corrections must be applied for the exponential atmosphere. The extended Sommerfeld theory is useful out to distances of order 10 $\lambda^{1/3}$ (km) and heights of the order of 35 $\lambda^{2/3}$ m where λ is the wavelength in metres.

2.2 *The line-of-sight region*

In the line-of-sight region, the field is composed of the sum of the direct and reflected waves. These are found from an integral representation by the method of stationary phase which is equivalent to geometrical optics. This differs from the linear atmosphere in that the points of stationary phase and the phase integrals must be evaluated numerically.

2.3 *The diffraction region*

The diffraction region occurs at ranges greater than about 10 $\lambda^{1/3}$ km (see § 2.1) near the surface or beyond the radio horizon for elevated terminals. Here the field can be represented by the residue or waveguide mode series. Associated with each mode is an eigenvalue, the propagation constant, and an eigenfunction, the height-gain function. The propagation constant determines the variation of the mode with range. The height-gain function determines the variation of the mode with height. The eigenfunction is the solution of a second order ordinary differential equation, and the eigenvalue is found by imposing the boundary conditions. There is an infinity of modes, but only a few are needed. Up to nine modes are used. An exact earth flattening method, Langer's transformation [1937] is employed. For the models in expressions (1) and (2) above, the solutions are known in closed form in terms of Airy functions. For the exponential model, the solutions are not expressible in terms of standard functions. The WKB Airy integral function [Bremmer, 1960 and 1962] approximations are used to give initial approximations to the eigenvalues. The Riccati differential equation of the wave impedance, or wave reflection coefficient, is then integrated numerically, and the eigenvalue found by Newton's method. The height-gain function is then found by numerical integration.

2.4 *Differences in propagation in linear and exponential atmospheres*

Under some conditions, the value of ground-wave field strength is insensitive to the model of refractive profile used. Under other conditions, the results between a linear profile and an exponential profile can differ by more than 20 dB. In some of these cases, the simpler linear profile computations [Kirby and Hughes, 1989] can be adjusted to approximate closely the exponential profile results.

When both terminals are near the surface, i.e. when the terminal heights are less than $5f^{-2/3}$ km for a frequency of f MHz, [Millington, 1958], both profiles give the same value of field strength except at distances beyond about 200 km. At 100 MHz, the difference in the computed field strength is about 1 dB at 500 km. This difference increases with increasing wavelength. It can be shown [Rotheram, 1981] that even at these longer wavelengths, the linear profile calculations can be made to agree with exponential profile calculations at long distances if the effective earth's radius factor is adjusted to a value which is less than the nominal value of 4/3. The appropriate value of this modified effective Earth's radius depends on the wavelength and, to a lesser extent, on the value of the conductivity of the Earth. Its value varies between the nominal value of 4/3 at 30 MHz and asymptotically approaches a value of 1 at 10 kHz [Rotheram 1970, Gerks 1971].

When one or both terminals are at a greater height, the linear profile approximation fails. The linear profile overestimates both the field strength and the distance from a terminal to the radio horizon. For an antenna at a height of 10 km, the error in the estimate of the distance to the radio horizon is about 11 km [Millington, 1958]. If the values of the field strength as a function of distance are computed with both profiles, it is found that the linear profile results can be made to agree with the exponential profile results at long distances if the linear profile curve is shifted by an amount equal to the difference in the radio horizon distances.

3. The computer program

A program GRWAVE ——————— has been developed using the methods outlined above [Rotheram, 1981]. The constants n_s and h_s in expression (3), the ground constants of the Earth, the radio frequency and the polarization are supplied as input data. ———————

——————— The field strength is calculated for all pairs of heights at all ranges. Of course only one eigenvalue and one integration of the height-gain differential equation suffices for each mode. A number of codes are input to control the mode of operation and the amount of output. The main output is the field strength at each point. This is given in µV/m for a transmitter of dipole moment 5 $\lambda/2\ \pi$ (amp-metres) giving a field strength of 1.5×10^5 µV/m at 1 km, which is the free-space field from a 3/4 kW isotropic radiator (which is the same as 1.732×10^5 µV/m at 1 km from a 1 kW isotropic radiator). The transmission loss and field strength relative to free space are also output. Within the line-of-sight region information about the interference lobe structure is given. By using a code, other output can be supplied, such as the propagation constants and height-gain functions, or very detailed output for error diagnosis.

Note. – The program is available from the CCIR Secretariat, for use by administrations.

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